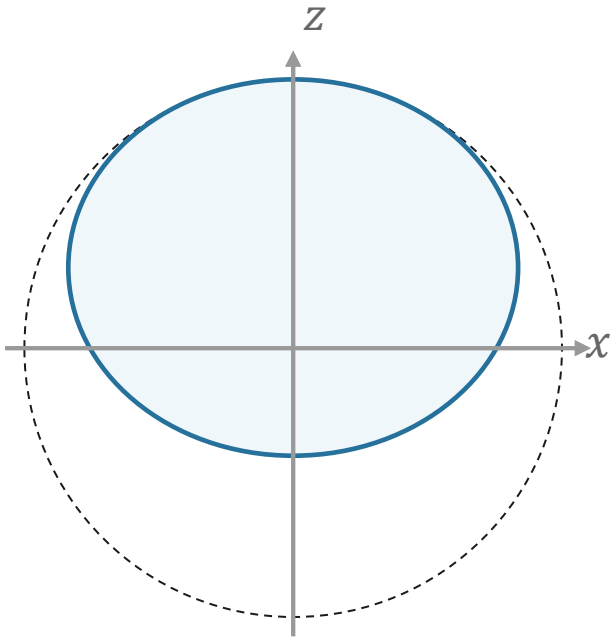
SAJID Lab
Smart & Advanced Junction of Intelligent Devices

EEE 6516 · QUANTUM COMPUTING

# Channels and Noise

Kraus operators, $T_1$ and $T_2$, readout error and noisy simulation

Chapter 7 · Lecture 16




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# Today




**By the end of this lecture you can**

- verify the completeness of a set of Kraus operators and describe what standard channels do to the Bloch ball
- relate γ, λ and p to $T_1, T_\varphi$ and $T_2$; explain $T_2 \leq 2T_1$ and $T_2^*$ against $T_2$
- model readout error with a confusion matrix and correct it by inversion
- build Aer noise models and a QuTiP master-equation run, and compare with analytic predictions

YOUR TURN · RECALL

# Recap: where is this state in the Bloch ball?

**Recall · 30 seconds**

Find the Bloch vector of $\rho = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.7 \end{bmatrix}$ using $r_z = \mathrm{Tr}(\rho\, Z)$.

**A** $r = (0,0,-0.4)$

**B** $r = (0,0,0.4)$

**C** $r = (0,0,-0.7)$

**D** outside the ball: not a state

ANSWER

# A: r = (0, 0, −0.4), inside the ball, below the centre

$r_z = \mathrm{Tr}(\rho\, Z) = \rho_{00} - \rho_{11} = 0.3 - 0.7 = -0.4; \quad r_x = r_y = 0$

Length 0.4 < 1: a mixed state, closer to |1⟩ than to |0⟩.

Today: noise moves Bloch vectors like this — shorter, and shifted.

# 1 Quantum channels

What is the most general thing noise can do to $\rho$?

# From a noisy bit to a noisy qubit

### Binary symmetric channel (classical)

$$\begin{bmatrix} P_0' \\ P_1' \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \end{bmatrix}$$

### Quantum channel, Kraus form

$$\mathcal{E}(\rho) = \sum_i K_i \, \rho \, K_i^\dagger$$

$$\sum_i K_i^\dagger K_i = I \quad \text{(completeness)}$$

Classically, noise is a stochastic matrix: its columns are probability vectors.

A quantum channel must also act on the coherences, and output a density matrix.

The general form: Kraus operators $K_i$ with $\Sigma K_i \dagger K_i = I$.

A unitary gate is the channel with one Kraus operator, U.

DERIVATION

# Why completeness keeps ρ a state

$$\mathrm{Tr}\,\mathcal{E}(\rho) = \sum_i \mathrm{Tr}\left(K_i \rho\, K_i^\dagger\right)$$ trace is linear

$$= \mathrm{Tr}\left(\sum_i K_i^\dagger\, K_i\, \rho\right) = \mathrm{Tr}\rho = 1$$ cyclic property, then completeness

$$\langle v|K_i \rho\, K_i^\dagger|v\rangle = \langle w|\rho|w\rangle \geq 0, \qquad |w\rangle = K_i^\dagger|v\rangle$$ each term is positive

Every CPTP map has a Kraus form (stated without proof: Kraus, Choi; N&C §8.2.3).

YOUR TURN · VOTE

# Quick vote: the condition on Kraus operators

**Vote · 30 seconds**

Which condition must the Kraus operators $K_i$ of a channel satisfy?

**A** $\sum_i K_i = I$

**B** $\sum_i K_i K_i^\dagger = 0$

**C** each $K_i$ is unitary

**D** $\sum_i K_i^\dagger K_i = I$

# Worked example: amplitude damping

**Amplitude damping, $0 \le \gamma \le 1$**

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \qquad K_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

$$K_0^\dagger K_0 + K_1^\dagger K_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1-\gamma \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \gamma \end{bmatrix} = I$$

1 Completeness: $K_0\dagger K_0 = \mathrm{diag}(1, 1-\gamma)$, $K_1\dagger K_1 = \mathrm{diag}(0, \gamma)$; the sum is I.

**Before the next slide**

What does the channel do to $|1\rangle$?

# Worked example: amplitude damping

**Amplitude damping, $0 \le \gamma \le 1$**

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \qquad K_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

$$K_0^{\dagger} K_0 + K_1^{\dagger} K_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1-\gamma \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \gamma \end{bmatrix} = I$$

$$\mathcal{E}(|1\rangle\langle 1|) = (1-\gamma)|1\rangle\langle 1| + \gamma|0\rangle\langle 0| = \begin{bmatrix} \gamma & 0 \\ 0 & 1-\gamma \end{bmatrix}$$

**1** Completeness: $K_0\dagger K_0 = \mathrm{diag}(1, 1-\gamma)$, $K_1\dagger K_1 = \mathrm{diag}(0, \gamma)$; the sum is I.

**2** $|1\rangle$ decays to $|0\rangle$ with probability γ. For γ = 0.3: diag(0.3, 0.7) — the recap matrix, r = (0, 0, −0.4).

**Before the next slide**

And to $|+\rangle$?

# Worked example: amplitude damping

**Amplitude damping, $0 \le \gamma \le 1$**

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}, \qquad K_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

$$K_0^\dagger K_0 + K_1^\dagger K_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1-\gamma \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \gamma \end{bmatrix} = I$$

$$\mathcal{E}(|1\rangle\langle 1|) = (1-\gamma)|1\rangle\langle 1| + \gamma|0\rangle\langle 0| = \begin{bmatrix} \gamma & 0 \\ 0 & 1-\gamma \end{bmatrix}$$

$$\mathcal{E}(|+\rangle\langle +|) = \frac{1}{2}\begin{bmatrix} 1+\gamma & \sqrt{1-\gamma} \\ \sqrt{1-\gamma} & 1-\gamma \end{bmatrix}$$

**1** Completeness: $K_0\dagger K_0 = \mathrm{diag}(1, 1-\gamma)$, $K_1\dagger K_1 = \mathrm{diag}(0, \gamma)$; the sum is I.

**2** $|1\rangle$ decays to $|0\rangle$ with probability γ. For $\gamma = 0.3$: diag(0.3, 0.7) — the recap matrix, $r = (0, 0, -0.4)$.

**3** $|+\rangle \mapsto r = (\sqrt{(1-\gamma)}, 0, \gamma)$; for $\gamma = 0.3$, (0.8367, 0, 0.3): shorter, and pushed towards $|0\rangle$.

# 2 Five standard noise models

What does each channel do to the Bloch ball?

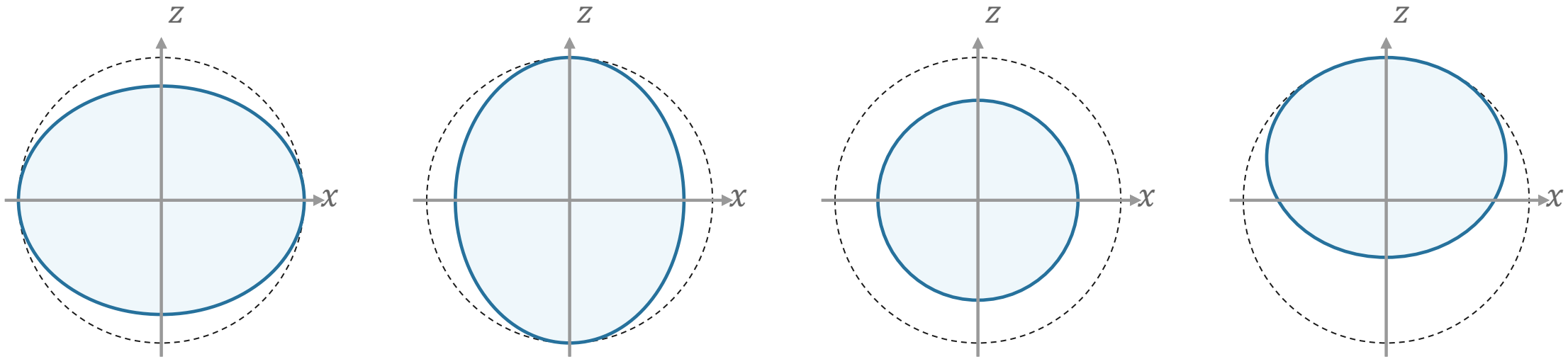
# Five channels in one table

| channel | Kraus operators / form | Bloch map $(x, y, z) \mapsto$ |
|---|---|---|
| bit flip | $\sqrt{(1-p)}\, I,\ \sqrt{p}\, X$ | $(x, (1-2p)y, (1-2p)z)$ |
| phase flip | $\sqrt{(1-p)}\, I,\ \sqrt{p}\, Z$ | $((1-2p)x, (1-2p)y, z)$ |
| depolarising | $(1-p)\rho + p\, I/2$ | $(1-p)\, r$ |
| amplitude damping | WE 7.5 ($\gamma$) | $(\sqrt{(1-\gamma)}x, \sqrt{(1-\gamma)}y, \gamma+(1-\gamma)z)$ |
| phase damping | $\mathrm{diag}(1, \sqrt{(1-\lambda)}),\ \mathrm{diag}(0, \sqrt{\lambda})$ | $(\sqrt{(1-\lambda)}x, \sqrt{(1-\lambda)}y, z)$ |

$p$ is the probability of an error; phase damping equals a phase flip with $1 - 2p = \sqrt{(1 - \lambda)}$.

Two different sets of Kraus operators can describe one channel.

# Four channels on the Bloch ball




bit flip, p = 0.1 | phase flip, p = 0.1 | depolarising, p = 0.3 | amp. damping, γ = 0.3

Shrink factors from Simulation 7.4: flips 0.8, depolarising 0.7, amplitude damping 0.8367 with a shift of 0.3 towards $|0\rangle$.

YOUR TURN · PREDICT

# Predict: “depolarising, p = 0.1” on |0⟩

**Think · 45 seconds**

Aer's `depolarizing_error(0.1, 1)` and Cirq's `depolarize(0.1)` act on |0⟩. What ⟨z⟩ does each give?

**A** both 0.9

**B** Aer 0.9, Cirq ≈ 0.867

**C** Aer ≈ 0.867, Cirq 0.9

**D** both 0.8

ANSWER

# B: Aer gives 0.9, Cirq (and PennyLane) ≈ 0.867

$$(1-p)\rho + p\frac{I}{2} = (1-q)\rho + \frac{q}{3}(X\rho X + Y\rho Y + Z\rho Z), \qquad q = \frac{3}{4}p$$

Aer's p is the replacement probability: shrink factor 1 − p = 0.9.

Cirq's and PennyLane's p is the probability q that X, Y or Z is applied: shrink 1 − 4q/3 ≈ 0.867.

Always state which parametrisation a depolarising p refers to.

YOUR TURN · VOTE

# Quick vote: Aer's depolarising error with p = 0.3

**Vote · 30 seconds**

Qiskit Aer's `depolarizing_error(0.3, 1)` acts on $|0\rangle$. What is the z component of the output Bloch vector?

**A** 0.3

**B** 0.6

**C** 0.7

**D** 0.85

COMMON MISTAKE

# “A slightly wrong rotation is decoherence”

## ✗ Over-rotation by ε = noise

$R(\theta + \varepsilon)$ instead of $R(\theta)$

'the qubit decohered a little'

Still a unitary: pure stays pure.

## ✓ Coherent error ≠ channel

coherent: undo by rotating back by ε

decoherence: needs > 1 Kraus operator; no unitary restores |r|

Diagnosed and corrected differently.

Both lower fidelity; only decoherence shortens the Bloch vector.

# 3 Relaxation and dephasing: $T_1$ and $T_2$

What happens to a qubit while it waits?

# $T_1$: energy loss; $T_\varphi$: phase loss

### Energy relaxation

$$\Pr(1)(t) = e^{-t/T_1}, \qquad \gamma(t) = 1 - e^{-t/T_1}$$

### Pure dephasing

$$\sqrt{1-\lambda(t)} = e^{-t/T_\varphi} \quad \text{(coherences only)}$$

$|0\rangle$ is the ground state: $|1\rangle$ gives up its energy, as an RC circuit discharges.

Two waits multiply survival factors, so $\gamma(t) = 1 - e^{-t/T_1}$ is consistent.

Amplitude damping also multiplies coherences by $\sqrt{1-\gamma} = e^{-t/2T_1}$.

Frequency fluctuations randomise the phase without exchanging energy: pure dephasing, time $T_\varphi$.

DERIVATION

# Both processes act: the $T_2$ relation

$$e^{-t/2T_1}\, e^{-t/T_\varphi} = e^{-t/T_2}$$

coherence factors multiply

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\varphi}$$

compare exponents

$$T_2 \leq 2T_1$$

equality only without pure dephasing

Equator state: moves in towards the z axis with $T_2$ while z relaxes to +1 with $T_1$.

YOUR TURN · PAIRS

# Your turn: a qubit with $T_1 = 100\ \mu s$, $T_\varphi = 120\ \mu s$

**Pairs · 1 minute**

Find $T_2$. Then: prepared in $|1\rangle$, what is Pr(1) after 30 μs?

**A** $T_2 = 75\ \mu s$;  $Pr(1) \approx 0.741$

**B** $T_2 = 220\ \mu s$;  $Pr(1) \approx 0.741$

**C** $T_2 = 54.5\ \mu s$;  $Pr(1) \approx 0.70$

**D** $T_2 = 75\ \mu s$;  $Pr(1) \approx 0.30$

ANSWER

# A: $T_2$ = 75 µs, Pr(1) = $e^{-0.3} \approx 0.741$

$$\frac{1}{T_2} = \frac{1}{200} + \frac{1}{120} = \frac{8}{600} \Rightarrow T_2 = 75\ \mu s < 2T_1$$

$\Pr(1) = e^{-30/100} \approx 0.741$; from |+⟩: $\langle X \rangle = e^{-30/75} \approx 0.670$

Then $z = 1 - e^{-0.3} \approx 0.259$, so Pr(0) ≈ 0.630, Pr(1) ≈ 0.370 for |+⟩.

Measure $T_1$ by prepare-|1⟩-wait-measure; $T_2$ with a Hahn echo; $T_2^*$ with a Ramsey sequence (next).

YOUR TURN · VOTE

# Quick vote: an impossible $T_2$

**Vote · 30 seconds**

A qubit has $T_1 = 50\ \mu s$. Which value of $T_2$ is impossible?

**A** 30 μs

**B** 80 μs

**C** 100 μs

**D** 120 μs

YOUR TURN · VOTE

# Quick vote: why does the echo last longer?

**Vote · 30 seconds**

Why is the decay time measured with a Hahn echo usually longer than the one measured with a Ramsey sequence?

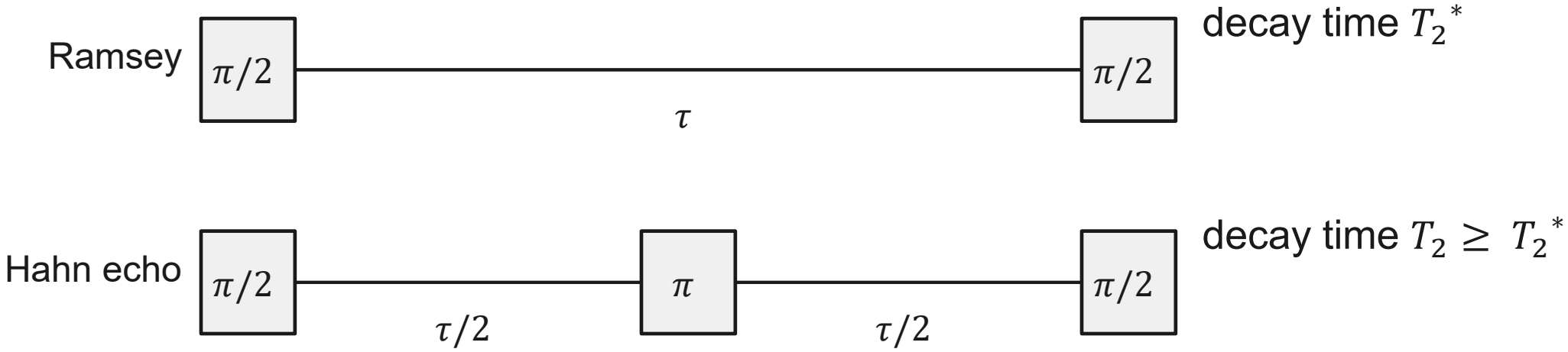
**A** the echo measures $T_1$, not a dephasing time

**B** the echo cancels slowly varying detuning

**C** the echo's π pulse lengthens $T_1$

**D** the echo averages over more shots

# Ramsey measures $T_2^*$, an echo measures $T_2$




Slow frequency drift: constant within one shot, different between shots; averaging washes the Ramsey fringes out faster than $T_2$ predicts.

The π pulse halfway reverses the phase accumulated so far: a constant detuning δ adds δτ/2, then cancels it. Only fast fluctuations survive.

# 4 Readout error and noisy simulation

What if the meter itself lies?

# Readout error is a classical channel on the outcome

### Confusion matrix (column-stochastic)

$$p_{\text{meas}} = A\, p_{\text{true}},$$
$$A_{rt} = \Pr(\text{report } r \mid \text{true } t)$$

### Correction

$$p_{\text{true}} \approx A^{-1} p_{\text{meas}}$$

Like the detection errors of a digital receiver: the qubit was |0⟩ but the meter reports 1.

Measure A beforehand: prepare each basis state, record what is reported.

A acts after the measurement: corrected in post-processing, not by a gate.

Qiskit Aer's ReadoutError takes $A^{T}$: its rows are the true outcomes.

YOUR TURN · PREDICT

# Your turn: what does the meter report for |+⟩?

**Think · 45 seconds**

Pr(report 1 | true 0) = 0.03 and Pr(report 0 | true 1) = 0.05.

The qubit is in |+⟩. What is the reported Pr(1)?

**A** 0.49

**B** 0.50

**C** 0.51

**D** 0.46

ANSWER

# A: 0.03 × ½ + 0.95 × ½ = 0.49

$$A = \begin{bmatrix} 0.97 & 0.05 \\ 0.03 & 0.95 \end{bmatrix}, \qquad A^{-1}\begin{bmatrix} 0.51 \\ 0.49 \end{bmatrix} = \frac{1}{0.92}\begin{bmatrix} 0.46 \\ 0.46 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Simulation 7.7, 20000 shots: measured Pr(1) = 0.4917; corrected (0.4982, 0.5018).

Binomial σ of the count ≈ √(20000 × 0.49 × 0.51) ≈ 71: within sampling error of (0.5, 0.5).

DERIVATION

# A Bell pair from a noisy CNOT

$$\rho = (1-p)|\Phi^+\rangle\langle\Phi^+| + p\,\frac{I}{4}$$

Aer's depolarizing_error(p, 2) after the CNOT

$$F = \langle\Phi^+|\rho|\Phi^+\rangle = (1-p) + \frac{p}{4}$$

⟨Φ⁺|I/4|Φ⁺⟩ = ¼

$$= 1 - \frac{3}{4}p$$

eq:bell-fidelity

p = 0.1 gives F = 0.925: the state of WE 7.4(b) from Lecture 15.

LIVE DEMO

# Simulation 7.8 (live): a Bell pair from a noisy CNOT

```
qc.h(0)
qc.cx(0, 1)
err = depolarizing_error(p, 2)
nm.add_all_qubit_quantum_error(err, ["cx"])
sim = AerSimulator(method="density_matrix",
```

The density-matrix method propagates ρ exactly: no shots, no sampling noise.

state_fidelity compares the output with $|\Phi^+\rangle$ (squared convention).

Comment on the err line omitted: it is (1 − p)ρ + p I/4, the previous slide.

| p | F (Aer) | 1 − 3p/4 |
|---|---|---|
| 0.00 | 1.000000 | 1.000000 |
| 0.01 | 0.992500 | 0.992500 |
| 0.05 | 0.962500 | 0.962500 |
| 0.10 | 0.925000 | 0.925000 |

# Grover's algorithm with noisy CNOTs

| k | CNOTs | ideal | noisy |
|---|---|---|---|
| 0 | 0 | 0.1250 | 0.1250 |
| 1 | 12 | 0.7813 | 0.6480 |
| 2 | 24 | 0.9453 | 0.6518 |
| 3 | 36 | 0.3301 | 0.2365 |
| 4 | 48 | 0.0122 | 0.0856 |

n = 3, marked $|101\rangle$; transpiled to CNOT, Rz, √X, X; 2 % depolarising error on every CNOT.

Optimum still k = 2, but its advantage nearly vanishes: 0.6480 at k = 1 against 0.6518 at k = 2.

Noisy simulations show trends and budgets. They are not hardware evidence: real devices add crosstalk, leakage, correlated noise and drift.

COMMON MISTAKE

# QuTiP's sigmam() and the excited state

## ✗ c_ops = [sqrt(1/T1) * sigmam()]

sigmam() maps basis(2, 0) to basis(2, 1)

it treats |0⟩ as the excited state

The qubit 'decays' into |1⟩.

## ✓ c_ops = [sqrt(1/T1) * destroy(2)]

$\mathrm{destroy}(2) = |0\rangle\langle 1|$

lowers |1⟩ to |0⟩: this book's decay

basis(2, 0) has ⟨Z⟩ = +1, as |0⟩ does here.

Check the convention of any library before using its named operators (QuTiP 5.3.1, checked for this course).

SIMULATION

# Simulation 7.6: $T_1$ and $T_2$ from the master equation

```
a = qt.destroy(2)                          # |0><1|: lowers |1> to |0>
c_ops = [np.sqrt(1 / T1) * a,
         np.sqrt(1 / (2 * Tphi)) * qt.sigmaz()]
```

Lindblad equation: $\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_k \left( L_k \rho\, {L_k}^\dagger - \frac{1}{2} {L_k}^\dagger L_k \rho - \frac{1}{2} \rho\, {L_k}^\dagger L_k \right)$

| t | Pr(1) | $e^{-t/T_1}$ | ⟨X⟩ | $e^{-t/T_2}$ |
|---|---|---|---|---|
| 30 μs | 0.740818 | 0.740818 | 0.670320 | 0.670320 |
| 75 μs | 0.472367 | 0.472367 | 0.367880 | 0.367879 |
| 100 μs | 0.367880 | 0.367879 | 0.263597 | 0.263597 |
| 200 μs | 0.135336 | 0.135335 | 0.069484 | 0.069483 |

# Recap

1

## Channels

$\mathcal{E}(\rho) = \sum_i K_i \rho\, K_i^\dagger, \sum_i K_i^\dagger K_i = I$; pictures in the Bloch ball

2

## Two meanings of depolarising p

Aer: replacement probability; Cirq/PennyLane: Pauli probability

3

## $T_1$, $T_2$, $T_2^*$

$1/T_2 = 1/2T_1 + 1/T_\varphi, T_2 \le 2T_1$; Ramsey gives $T_2^*$, echo $T_2$

4

## Readout and noisy simulation

invert the confusion matrix; Bell F = 1 − 3p/4; trends, not hardware

# Next week: Lecture 17 (Chapter 8)

## Error correction: can we undo a channel we do not know?

Read 7.5–7.7; self-study 7.8; Problems PRB-0190, PRB-0191.

**Exit ticket**

A bit-flip channel with $p = 0.1$ acts on $|0\rangle$ and on $|+\rangle$. Give both Bloch vectors and both fidelities to the input. Which input is safe, and why?

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