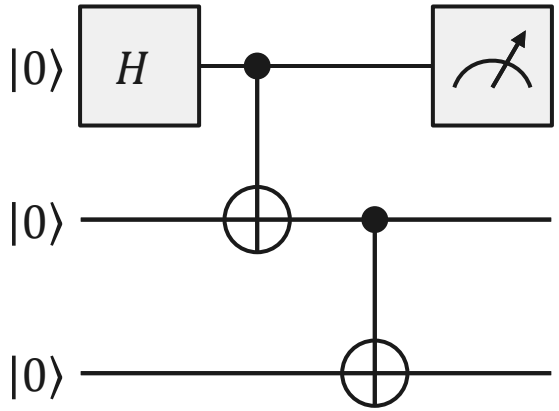
SAJID Lab
Smart & Advanced Junction of Intelligent Devices

EEE 6516 · QUANTUM COMPUTING

The Circuit Model

Measurement, reversible logic, universality and transpilation

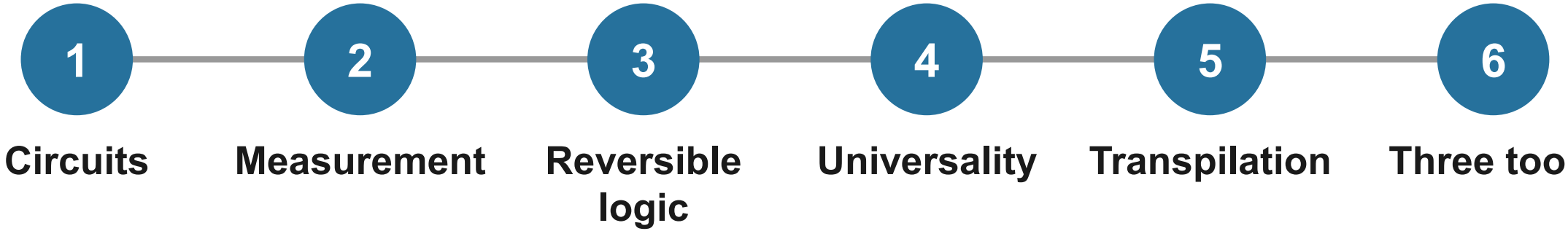
Chapter 3 · Lecture 7



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Today



By the end of this lecture you can

- compute the state and outcome probabilities of a multi-qubit circuit, including measuring one qubit
- apply the principle of deferred measurement
- build AND and NAND reversibly with Toffoli gates; state which gate sets are universal
- read transpiler resource counts; run one circuit in Qiskit, Cirq and PennyLane

YOUR TURN · RECALL

Recap: CNOT on a basis state

Recall · 30 seconds

What is $\mathrm{CNOT}_{01}|11\rangle$ (control qubit 0, target qubit 1)?

A $|00\rangle$

B $|01\rangle$

C $|10\rangle$

D $|11\rangle$

ANSWER

C: the control is 1, so the target flips

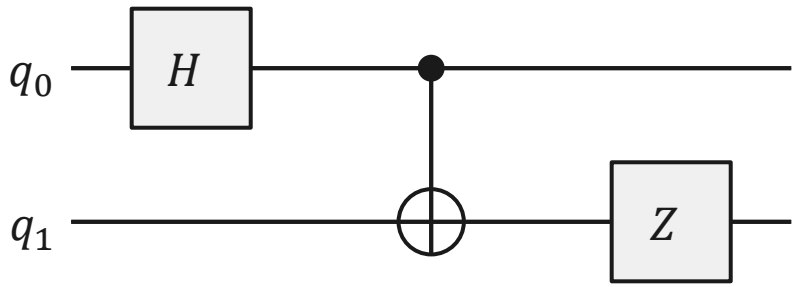
$|a\rangle|b\rangle \to |a\rangle|b \oplus a\rangle$: $|11\rangle \to |1\rangle|1 \oplus 1\rangle = |10\rangle$

Today: whole circuits, with measurements, and what it costs to run them.

1 The circuit model

How does a circuit become one matrix, and what does it cost?

A circuit is a product of matrices



$$U = (I \otimes Z)\,\mathrm{CNOT}_{01}\,(H \otimes I)$$

matrices in reverse time order; I on idle qubits

Time runs left to right; qubit 0 is the top wire.

Each gate becomes a $2^n \times 2^n$ matrix with I on the qubits it does not touch.

The circuit's unitary is the product in reverse time order.

Three numbers describe the cost

width
number of qubits

gate count
single- and two-qubit gates counted separately

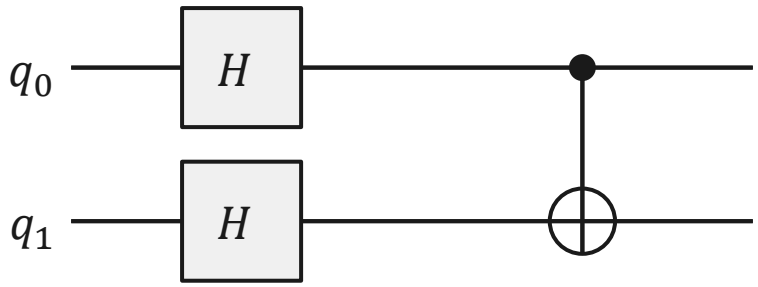
depth
number of layers; gates on disjoint qubits share a layer

H then CNOT on two qubits: width 2, two gates (one two-qubit), depth 2.

The two kinds of gate have different costs on hardware (Chapter 9 gives dated figures).

YOUR TURN · CALCULATE

Your turn: count the cost



Think · 45 seconds

H on both qubits, then a CNOT.

What are the width, the gate count and the depth?

ANSWER

Width 2, three gates, depth 2

The two H gates act on different qubits: one layer.

The CNOT needs both qubits: a second layer.

Gate count 3, of which one two-qubit gate.

2 Measurement inside circuits

What happens to qubit 1 when only qubit 0 is measured?

Measuring one qubit of several

Measuring qubit 0 of two

$$\Pr(k) = \sum_{q_1} \left| a_{kq_1} \right|^2$$

$$|\psi\rangle \to \frac{1}{\sqrt{\Pr(k)}} \sum_{q_1} a_{kq_1} |k\rangle |q_1\rangle$$

Add the weights of the basis states whose first label is k.

Keep the consistent terms; renormalise.

Qubit 1 is not measured, but its state may change.

Worked example: measuring one qubit of two

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$$

$$\Pr(0) = \frac{2}{3} : \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) = |0\rangle \otimes |+\rangle$$

1 Outcome 0: the terms with q0 = 0.

Before the next slide

And outcome 1?

Worked example: measuring one qubit of two

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$$

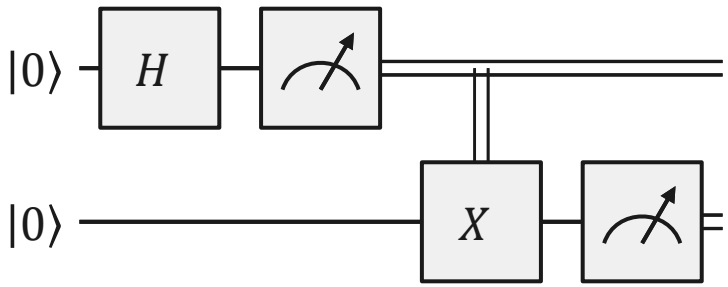
$$\Pr(0) = \frac{2}{3}: \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle) = |0\rangle \otimes |+\rangle$$

$$\Pr(1) = \frac{1}{3}: |10\rangle$$

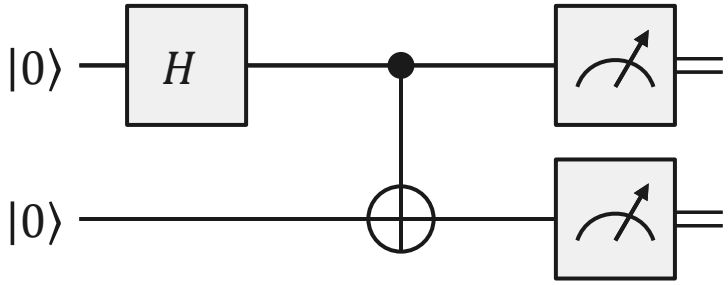
1 Outcome 0: the terms with q0 = 0.

2 Qubit 1 ends in $|+\rangle$ or $|0\rangle$ depending on qubit 0: a correlation from entanglement.

Deferred measurement



(a) measure, then classically controlled X



(b) CNOT, then measure

A measurement that only controls later gates can be moved to the end.

Replace the classical control by a quantum control.

Both give 00 or 11, each with probability ½.

It does not let measurements commute with arbitrary gates.

SIMULATION

Simulation: deferred measurement

```
a.measure(0, 0)
with a.if_test((a.clbits[0], 1)):
    a.x(1)
b.cx(0, 1)
b.measure([0, 1], [0, 1])
```

if_test implements the classically controlled X.

Counts converted to textbook order.

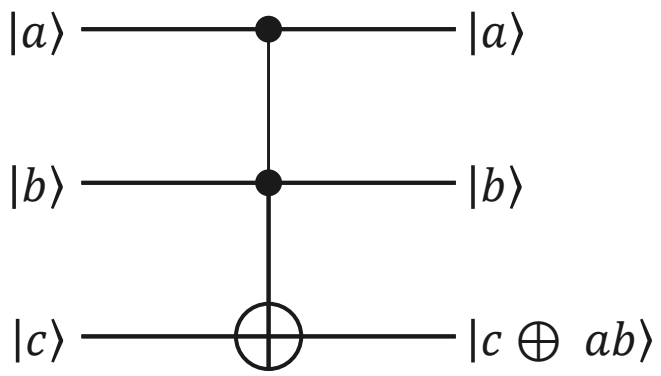
circuit	00	11
(a) measure, then if	998	1002
(b) CNOT, then measure	980	1020

2000 shots each; nothing else occurs; $\sigma \approx 22$

3 Reversible logic and the Toffoli gate

How can a unitary computer compute AND?

Toffoli: a reversible AND



$c = 0$: ab (AND); $\quad c = 1$: $\overline{ab}$ (NAND)

AND loses information: output 0 could come from 00, 01 or 10.

Toffoli keeps the inputs and is its own inverse.

With ancillas it simulates any classical circuit (Nielsen & Chuang §1.4.1, §3.2.5).

YOUR TURN · CALCULATE

Your turn: Toffoli on basis states

Think · 1 minute

Controls qubits 0 and 1, target qubit 2. Apply the Toffoli to $|110\rangle, |101\rangle, |111\rangle$.

Which input computes AND(1, 1)?

ANSWER

$|110\rangle \to |111\rangle$, $|101\rangle$ unchanged, $|111\rangle \to |110\rangle$

Both controls 1: the target flips. One control 0: nothing happens.

$|110\rangle$ has target 0 and leaves $1 \cdot 1 = 1$: AND(1, 1).

Only $|110\rangle$ and $|111\rangle$ change, and they exchange.

COMMON MISTAKE

“An m-gate Boolean circuit fits on $\log_2 m$ qubits”

✗ Exponential space, logarithmic qubits

2^n amplitudes store the whole circuit

so $\log_2 m$ qubits suffice

Amplitudes cannot be read out as bits.

✓ One qubit per wire, plus ancillas

keep every input and intermediate value

until it is uncomputed

measuring n qubits gives n bits (Ch. 1)

Of the order of the wires and gates.

Reversible simulation needs at least one qubit per input wire plus ancillas, not a logarithm of the gate count.

4 Universal gate sets

Which few gates are enough to build every unitary?

Exact and approximate universality

Exact (Nielsen & Chuang §4.5.2)

$$\text{CNOT} + \text{every gate on one qubit}$$

Approximate (Nielsen & Chuang §4.5.3)

$$\{H, T, \text{CNOT}\}\text{: } O\left(\log^c(1/\varepsilon)\right) \text{ gates}, c \approx 2$$

$\{H, S, \text{CNOT}\}$: not universal (Clifford only)

Single-qubit gates form a continuum: an infinite set.

A finite set can only approximate, but cheaply: a 1000× smaller error costs about 4× more gates (c = 2).

Adding T to {H, S, CNOT} makes the set universal.

COMMON MISTAKE

“H and CNOT are a universal gate set”

✗ Seen in every example, so enough

H, CNOT (and S) build Bell states, teleportation, many demos

They generate only Clifford operations.

✓ Add a non-Clifford gate

add T or an arbitrary rotation

$\{H, T, \mathrm{CNOT}\}$ is approximately universal

Check for a non-Clifford gate.

A universal set needs a non-Clifford gate such as T; H, S and CNOT alone are not enough.

Why a finite set is practical

$$\text{gates needed per gate on one qubit} \;\propto\; \log^{c}(1/\varepsilon)$$

ε	$\log_2(1/\varepsilon)^2$
10^{-2}	≈ 44
10^{-4}	≈ 177
10^{-6}	≈ 397

The cost grows with the logarithm of the accuracy, not with 1/ε.

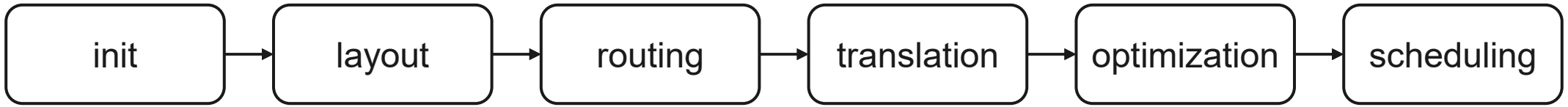
Table: the scaling $(\log_2 1/\varepsilon)^2$ for c = 2, constants omitted.

Illustration of the scaling only, not gate counts of a real compiler.

5 From circuit to hardware

What does a compiler do to a circuit before it runs?

Transpilation: basis gates and a coupling map



stage names as reported by Qiskit 2.5.2 (preset pass manager)

Basis gates: the few gates a processor offers.

Coupling map: which pairs of qubits a two-qubit gate may join.

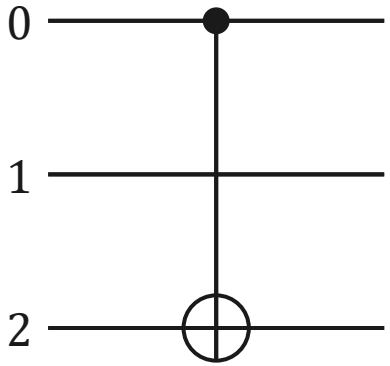
Layout places logical qubits; routing inserts SWAPs; translation rewrites into basis gates; optimisation simplifies.

The unitary is unchanged (up to a global phase and a recorded permutation); the cost is not.

YOUR TURN · PREDICT

Predict: a CNOT between the ends of a line





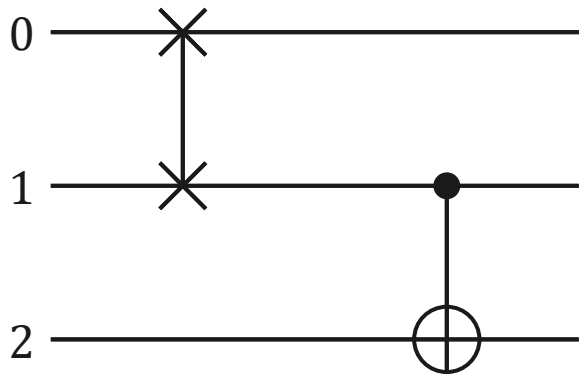
Predict · 1 minute

Three qubits on a line 0–1–2; two-qubit gates only between neighbours.

How many CNOTs does CNOT(0, 2) cost, and what is the depth?

ANSWER

4 CNOTs and depth 4 (against 1 and 1)



SWAP(0, 1) = 3 CNOTs, then CNOT(1, 2)

One SWAP moves qubit 0's state next to qubit 2: 3 CNOTs.

Then CNOT(1, 2) on a coupled pair: 4 in total, one after another.

The transpiler records the permutation instead of undoing it.

SIMULATION

Simulation: gate counts after transpilation

```
basis = ["cx", "h", "t", "tdg"]
qc.ccx(0, 1, 2)
qc.cx(0, 2)
n_cx = t.count_ops().get("cx", 0)
```

Toffoli: same counts at optimisation levels 0, 1 and 3.

Two-qubit count and depth are the numbers to watch: each adds time and error on a device.

circuit	CNOT	gates	depth
Toffoli	6	15	11
CNOT(0,2) all-to-all	1	1	1
CNOT(0,2) line	**4**	**4**	**4**

Qiskit 2.5.2; counts change between versions

6 One circuit, three simulators

Do Qiskit, Cirq and PennyLane agree?

Three tools, one physics

Qiskit
qubit 0 rightmost: convert with reverse_qargs()

Cirq
textbook order; pass qubit_order= or idle qubits vanish

PennyLane
textbook order: wire 0 is the leftmost bit

Cirq and PennyLane (introduced today) are used for comparison; Qiskit stays primary.

The code looks different; the numbers must not.

Test with an asymmetric circuit.

SIMULATION

Simulation: one circuit in three tools

```
    sv = Statevector(qc).reverse_qargs()
    q = cirq.LineQubit.range(2)
dev = qml.device("default.qubit", wires=2)
        qml.CNOT(wires=[0, 1])
    return qml.probs(wires=[0, 1])
```

H, CNOT: the symmetric state hides ordering errors.

X on q0: all probability on |10⟩ (index 2) in every tool.

circuit	tool	probabilities
H, CNOT	Qiskit	[0.5 0. 0. 0.5]
H, CNOT	Cirq	[0.5 0. 0. 0.5]
H, CNOT	PennyLane	[0.5 0. 0. 0.5]
X on q0	Qiskit	[0. 0. 1. 0.]
X on q0	Cirq	[0. 0. 1. 0.]
X on q0	PennyLane	[0. 0. 1. 0.]

Recap

1

Circuits

product of matrices; width, gate count, depth

2

Measurement

measuring one qubit keeps the consistent terms; deferred measurement

3

Toffoli and universality

reversible AND; $\{H, T, \mathrm{CNOT}\}$ approximately universal

4

Transpilation

routing adds SWAPs: more CNOTs, more depth

Next: Lecture 8 (Chapter 4)

Entanglement and Bell states

Read: handout 3.6–3.10; self-study 3.11.

Exit ticket

Measure qubit 1 (not qubit 0) of $\frac{1}{\sqrt{3}}(|00\rangle + |01\rangle + |10\rangle)$. Give the probabilities and the states afterwards.

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